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POWER-LAW BODIES OF MAXIMUM LIFT-TO-DRAG RATIO IN HYPERSONIC FLOW(*)

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ANGELO MIELE(**) and HO-YI HUANG(***)

SUMMARY

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The problem of maximizing the lift-to-drag ratio of a slender, flat-top, hypersonic body is investigated under the assumptions that the pressure distribution is Newtonian and the skin-friction coefficient is constant. Direct methods are employed, and the analysis is confined to the class of bodies whose transversal contour is semicircular and whose longitudinal contour is a power law.

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First, unconstrained configurations are considered, and the combination of power law exponent and thickness ratio maximizing the lift-to-drag ratio is determined. For a friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 3.6$ and corresponds to a conical configuration of thickness ratio $\tau = 0.118$.

Next, constrained configurations are considered, that is, conditions are imposed on the length, the thickness, the volume, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter.

Author

1. INTRODUCTION

In Ref. 1, the basic theory of slender, flat-top, homothetic bodies in the hypersonic regime was formulated under the assumptions that the pressure distribution is Newtonian and the skin-friction coefficient is constant. Analytical expressions were derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration, that is, the longitudinal and transversal contours of the homothetic body. In Ref. 2, two complementary variational problems were formulated, that of optimizing the longitudinal contour for a given transversal contour and that of optimizing the transversal contour for a given longitudinal contour, the criterion of optimization being the lift-to-drag ratio. The existence of similar solutions was investigated, and it was concluded that (1) the optimum longitudinal contour of a body of arbitrary transversal contour can be determined from the known optimum longitudinal contour of a body of semicircular cross section and (2) the optimum transversal contour of a body of arbitrary longitudinal contour can be determined from the known optimum transversal contour of a conical body.

The next step is to determine the extremal properties of these reference bodies. Here, a body of semicircular cross section is considered, and its longitudinal contour is determined for given constraints imposed on the length, the thickness, the volume, and the position of the center of pressure. Direct methods are employed, and the analysis is confined to the class of power law contours. In a subsequent report (Ref. 3), this limitation is removed, and the longitudinal contour is determined with the indirect methods of the calculus of variations.

The hypotheses employed are as follows: (a) a plane of symmetry exists between the left-hand and right-hand sides of the body; (b) the upper surface is flat; (c) the base plane is perpendicular to both the plane of symmetry and the plane of the flat top; (d) the free-stream velocity is perpendicular to the base plane and, therefore, is parallel to the line of intersection of the plane of symmetry and the plane of the flat top; (e) the pressure coefficient is twice the cosine squared of the angle formed by the free-stream velocity and the normal to each surface element; (f) the skin-friction coefficient is constant; (g) the contribution of the tangential forces to the lift is negligible with respect to the contribution of the normal forces; (h) the body is slender in the longitudinal sense; (i) the transversal contour is semicircular; and (j) the longitudinal contour is represented by a power law.

2. FUNDAMENTAL EQUATIONS

In order to relate the drag, the lift, and the pitching moment of a body to its geometry, we define two coordinate systems (Fig. 1): a Cartesian coordinate system Oxyz and a cylindrical coordinate system $Oxr\theta$. For the Cartesian coordinate system, the origin O is the apex of the body; the x-axis is the intersection of the plane of symmetry and the flat top, positive toward the base; the z-axis is contained in the plane of symmetry, perpendicular to the x-axis, and positive downward; and the y-axis is such that the xyz-system is right-handed. For the cylindrical coordinate system, r is the distance of any point from the x-axis, and θ measures the angular position of this point with respect to the xy-plane.

If the hypotheses (a) through (h) are invoked and if the lower surface is represented by the relationship

$$r = r(x, \theta) \tag{1}$$

the drag D, the lift L, and the pitching moment M per unit free-stream dynamic pressure q can be written as (Refs. 1 and 2)

$$D/q = \int_0^{\ell} \int_0^{\pi/2} \left[4r^3 r_x^3/(r^2 + r_\theta^2) + 2C_f \sqrt{r^2 + r_\theta^2} \right] dx d\theta + 2C_f \int_0^{\ell} r(x,0) dx$$

$$L/q = \int_0^{\ell} \int_0^{\pi/2} \left[4r^2 r_x^2/(r^2 + r_\theta^2) \right] (r \sin\theta - r_\theta \cos\theta) \, dx d\theta \tag{2}$$

$$M/q = \int_0^{\ell} \int_0^{\pi/2} \left[4xr^2 r_x^2/(r^2 + r_\theta^2) \right] (r \sin\theta - r_\theta \cos\theta) \, dx d\theta$$

where ℓ is the length of the body and where the subscripts x and θ denote partial derivatives. Next, we invoke hypothesis (i) and observe that, if every cross section is semicircular, the function (1) degenerates into

$$r = r(x) \tag{3}$$

with the implication that

$$r_\theta = 0 \tag{4}$$

everywhere. Hence, the surface integrals (2) reduce to the line integrals

$$D/q = \int_0^{\ell} r[2\pi\dot{r}^3 + C_f(2 + \pi)] \, dx$$

$$L/q = \int_0^{\ell} 4r\dot{r}^2 \, dx \tag{5}$$

$$M/q = \int_0^{\ell} 4xr\dot{r}^2 \, dx$$

in which $\dot{r}$ denotes the total derivative dr/dx. In accordance with hypothesis (j), we specialize the longitudinal contour (3) to the power law

$$r/t = (x/\ell)^n \tag{6}$$

in which n is an undetermined exponent and t is the base thickness. Consequently, Eqs. (5) become

$$\begin{aligned} D/q &= \ell^2 \tau \, (\tau^3 f_1 + C_f f_2) \\ L/q &= \ell^2 \tau^3 f_3 \\ M/q &= \ell^3 \tau^3 f_4 \end{aligned} \tag{7}$$

where

$$\tau = t/\ell \tag{8}$$

is the thickness ratio and where

$$\begin{aligned} f_1(n) &= \pi n^3/(2n - 1) \\ f_2(n) &= (2 + \pi)/(n + 1) \\ f_3(n) &= 4n^2/(3n - 1) \\ f_4(n) &= 4n/3 \end{aligned} \tag{9}$$

These equations are valid for $n > 1/2$ only, owing to the fact that the pressure drag cannot be negative.

Once the drag, the lift, and the pitching moment are known, certain derived quantities can be calculated. They are the lift-to-drag ratio E and the nondimensional distance ξ_o of the center of pressure from the apex. These quantities are defined by

$$E = L/D \quad , \qquad \xi_o = x_o/\ell = M/L\ell \tag{10}$$

and, because of Eqs. (7), can be rewritten as

$$E = \tau^2 f_3/(\tau^3 f_1 + C_f f_2) \ , \quad \xi_o = f_4/f_3 \tag{11}$$

Clearly, the lift-to-drag ratio depends on both the thickness ratio and the power law exponent, while the position of the center of pressure depends on the power law exponent only.

Finally, the volume of a flat-top, symmetric body is given by

$$V = \int_0^{\ell} \int_0^{\pi/2} r^2 dx d\theta \tag{12}$$

and reduces to

$$V = (\pi/2) \int_0^{\ell} r^2 dx \tag{13}$$

if the transversal contour is semicircular. In addition, if the longitudinal contour is represented by a power law, Eq. (13) simplifies to

$$V = \ell^3 \tau^2 f_5 \tag{14}$$

where

$$f_5(n) = \pi/2\ (2n + 1) \tag{15}$$

3. UNCONSTRAINED CONFIGURATION

The first step in the analysis is to determine the maximum lift-to-drag ratio of a configuration which is unconstrained geometrically and aerodynamically. According to Eq. (11-1), the lift-to-drag ratio depends on both the thickness ratio and the power law exponent, that is, it has the form $E = E(\tau, n)$. Therefore, the optimum values of τ and n are determined by the simultaneous relationships

$$E_\tau = 0 \quad , \quad E_n = 0 \tag{16}$$

in which the subscripts denote partial derivatives. These relationships can be written explicitly as

$$\begin{aligned} &\tau^3 f_1 - 2C_f f_2 = 0 \\ &\dot{f}_3(\tau^3 f_1 + C_f f_2) - f_3(\tau^3 \dot{f}_1 + C_f \dot{f}_2) = 0 \end{aligned} \tag{17}$$

with the dot sign denoting a total derivative with respect to n. From Eq. (17-1), it appears that the optimum thickness ratio is such that the skin-friction drag is one-third of the total drag. Furthermore, upon eliminating the thickness ratio from Eqs. (17), we obtain the relationship

$$2g_1 + g_2 - 3g_3 = 0 \tag{18}$$

where

$$g_1 = \dot{f}_1/f_1 \quad , \quad g_2 = \dot{f}_2/f_2 \quad , \quad g_3 = \dot{f}_3/f_3 \tag{19}$$

On account of the definitions (9-1) through (9-3), we see that Eq. (18) is solved by

$$n = 1 \tag{20}$$

which means that the optimum longitudinal contour is conical. With this understanding, the thickness ratio (17-1) and the lift-to-drag ratio (11-1) become

$$\begin{aligned} \tau C_f^{-1/3} &= (2/\pi + 1)^{1/3} \cong 1.178 \\ E C_f^{1/3} &= 4/3\pi^{2/3}(2 + \pi)^{1/3} \cong 0.3601 \end{aligned} \tag{21}$$

Equation (21-2) represents the upper limit to the lift-to-drag ratio which can be obtained with a flat-top configuration of semicircular cross section subjected to a flow parallel to the flat top. Should the configuration be required to satisfy a certain number of geometric and/or aerodynamic constraints, a loss in the lift-to-drag ratio would occur with respect to that predicted by Eq. (21-2).

4. GIVEN CENTER OF PRESSURE

To prescribe the nondimensional distance of the center of pressure from the apex is equivalent to prescribing the power law exponent in accordance with Eq. (11-2). Therefore, the lift-to-drag ratio can be maximized with respect to the thickness ratio only, and the relevant optimum condition is represented by Eq. (16-1) implicitly or Eq. (17-1) explicitly. Because of Eq. (17-1), the optimum thickness ratio is given by

$$\tau C_f^{-1/3} = (2f_2/f_1)^{1/3} \tag{22}$$

and the associated lift-to-drag ratio is

$$EC_f^{1/3} = (f_3/3)(4/f_1^2 f_2)^{1/3} \tag{23}$$

The parametric equations (11-2), (22), and (23) admit solutions of the form

$$n = P(\xi_o) \quad , \quad \tau C_f^{-1/3} = Q(\xi_o) \quad , \quad EC_f^{1/3} = R(\xi_o) \tag{24}$$

which are plotted in Figs. 2 through 4. For $\xi_o = 2/3$, the body is conical, and the maximum lift-to-drag ratio reaches the upper limit represented by Eq. (21-2). For any other value of ξ_o, lower values of the lift-to-drag ratio are obtained as shown in Fig. 4.

5. GIVEN THICKNESS AND LENGTH

To prescribe the thickness and the length is equivalent to prescribing the thickness ratio τ in accordance with the definition (8). Therefore, the lift-to-drag ratio (11-1) can be maximized with respect to the power law exponent only, and the relevant optimum condition is represented by Eq. (16-2) implicitly or Eq. (17-2) explicitly. Because of Eq. (17-2), the optimum power law exponent satisfies the relationship

$$\tau C_f^{-1/3} = \left(\frac{f_1}{g_2 - g_3}\right)^{-1/3} \left(\frac{f_2}{g_3 - g_1}\right)^{1/3} \tag{25}$$

whose solutions are such that $n > 0.789$. The associated lift-to-drag ratio is given by

$$EC_f^{1/3} = \left(\frac{f_1}{g_2 - g_3}\right)^{-2/3} \left(\frac{f_2}{g_3 - g_1}\right)^{-1/3} \left(-\frac{f_3}{g_1 - g_2}\right) \tag{26}$$

The parametric equations (25) and (26) admit solutions of the form

$$n = P(\tau C_f^{-1/3}) \quad , \quad EC_f^{1/3} = R(\tau C_f^{-1/3}) \tag{27}$$

which are plotted in Figs. 5 and 6. When the thickness-length parameter has the value 1.178, the configuration is conical, and the associated lift-to-drag ratio is $EC_f^{1/3} = 0.3601$. For larger values of the thickness-length parameter, the configuration is convex, and for smaller values, it is concave.

6. GIVEN VOLUME AND LENGTH

If the volume and the length are given, it is convenient to rewrite Eq. (14) in the form

$$V/\ell^3 = \tau^2 f_5 \tag{28}$$

The lift-to-drag ratio (11-1) is to be maximized with respect to the combinations of τ and n which ensure the constancy of the right-hand side of Eq. (28). In accordance with Lagrange multiplier theory, we introduce an undetermined constant λ and define the fundamental function

$$F = E + \lambda\tau^2 f_5 \tag{29}$$

Then, the optimum conditions are

$$F_\tau = 0 \quad , \quad F_n = 0 \tag{30}$$

which are equivalent to

$$E_\tau + 2\lambda\tau f_5 = 0 \quad , \quad E_n + \lambda\tau^2 \dot{f}_5 = 0 \tag{31}$$

and, upon elimination of the Lagrange multiplier, imply that

$$\tau g_5 E_\tau - 2E_n = 0 \tag{32}$$

where

$$g_5 = \dot{f}_5 / f_5 \tag{33}$$

In the light of Eq. (11-1), Eq. (32) can be rewritten as

$$\tau C_f^{-1/3} = \left(\frac{f_1}{g_2 - g_3 + g_5}\right)^{-1/3} \left(\frac{2f_2}{2g_3 + g_5 - 2g_1}\right)^{1/3} \tag{34}$$

and its solutions are such that $n > 0.735$. The associated lift-to-drag ratio and volume-length parameter are given by

$$\begin{aligned} EC_f^{1/3} &= \left(\frac{f_1}{g_2 - g_3 + g_5}\right)^{-2/3} \left(\frac{2f_2}{2g_3 + g_5 - 2g_1}\right)^{-1/3} \left(\frac{2f_3}{3g_5 - 2g_1 + 2g_2}\right) \\ V\ell^{-3} C_f^{-2/3} &= \left(\frac{f_1}{g_2 - g_3 + g_5}\right)^{-2/3} \left(\frac{2f_2}{2g_3 + g_5 - 2g_1}\right)^{2/3} f_5 \end{aligned} \tag{35}$$

The parametric equations (34) and (35) admit solutions of the form

$$n = P(V\ell^{-3}C_f^{-2/3}) \ , \quad \tau C_f^{-1/3} = Q(V\ell^{-3}C_f^{-2/3}) \ , \quad EC_f^{1/3} = R(V\ell^{-3}C_f^{-2/3}) \qquad (36)$$

which are plotted in Figs. 7 through 9. When the volume-length parameter has the value 0.7272, the configuration is conical, with a thickness ratio $\tau C_f^{-1/3} = 1.178$ and a lift-to-drag ratio $EC_f^{1/3} = 0.3601$. For larger values of the volume-length parameter, the configuration is convex, and for smaller values, it is concave.

7. GIVEN VOLUME AND THICKNESS

If the volume and the thickness are given, it is convenient to rewrite Eq. (14) in the form

$$V/t^3 = f_5/\tau \tag{37}$$

The lift-to-drag ratio (11-1) is to be maximized with respect to the combinations of τ and n which ensure the constancy of the right-hand side of Eq. (37). In accordance with Lagrange multiplier theory, we introduce an undetermined constant λ and define the fundamental function

$$F = E + \lambda f_5/\tau \tag{38}$$

Then, the optimum conditions are

$$F_\tau = 0 \quad , \qquad F_n = 0 \tag{39}$$

which are equivalent to

$$\tau^2 E_\tau - \lambda f_5 = 0 \quad , \qquad \tau E_n + \lambda \dot{f}_5 = 0 \tag{40}$$

and, upon elimination of the Lagrange multiplier, imply that

$$\tau E_\tau g_5 + E_n = 0 \tag{41}$$

In the light of Eq. (11-1), Eq. (41) can be rewritten as

$$\tau C_f^{-1/3} = \left(\frac{f_1}{g_2 - g_3 - 2g_5}\right)^{-1/3} \left(\frac{f_2}{g_3 - g_5 - g_1}\right)^{1/3} \tag{42}$$

and its solutions are such that $0.5 < n < 1.31$. The associated lift-to-drag ratio and volume-thickness parameter are given by

$$\begin{aligned} EC_f^{1/3} &= \left(\frac{f_1}{g_2 - g_3 - 2g_5}\right)^{-2/3} \left(\frac{f_2}{g_3 - g_5 - g_1}\right)^{-1/3} \left(-\frac{f_3}{3g_5 + g_1 - g_2}\right) \\ Vt^{-3} C_f^{1/3} &= \left(\frac{f_1}{g_2 - g_3 - 2g_5}\right)^{1/3} \left(\frac{f_2}{g_3 - g_5 - g_1}\right)^{-1/3} f_5 \end{aligned} \tag{43}$$

The parametric equations (42) and (43) admit solutions of the form

$$n = P(Vt^{-3} C_f^{1/3}) \;, \; \tau C_f^{-1/3} = Q(Vt^{-3} C_f^{1/3}) \;, \; EC_f^{1/3} = R(Vt^{-3} C_f^{1/3}) \tag{44}$$

which are plotted in Figs. 10 through 12. When the volume-thickness parameter has the value 0.4443, the configuration is conical, with a thickness ratio $\tau C_f^{-1/3} = 1.178$ and a lift-to-drag ratio $EC_f^{1/3} = 0.3601$. For larger values of the volume-thickness parameter, the configuration is convex, and for smaller values, it is concave.

8. DISCUSSION AND CONCLUSIONS

In the previous sections, the problem of maximizing the lift-to-drag ratio of a slender, flat-top, hypersonic body is investigated under the assumptions that the pressure distribution is Newtonian and the skin-friction coefficient is constant. Direct methods are employed, and the analysis is confined to the class of bodies whose transversal contour is semicircular and whose longitudinal contour is a power law.

First, unconstrained configurations are considered, and the combination of power law exponent and thickness ratio maximizing the lift-to-drag ratio is determined. For a friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 3.6$ and corresponds to a conical configuration of thickness ratio $\tau = 0.118$.

Next, constrained configurations are considered, that is, given conditions are imposed on the length, the thickness, the volume, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter. The lift-to-drag ratio of a constrained configuration is smaller than that of the optimum unconstrained configuration; however, for a particular value of the similarity parameter, equality is achieved.

While the longitudinal contour is conical for an unconstrained configuration, it can be either convex or concave for constrained configurations, depending on the value of the similarity parameter. Since the Newtonian pressure law has been verified experimentally for convex configurations only, the results pertaining to concave configurations are merely indicative of qualitative trends.

Finally, it is of interest to compare the present lift-to-drag ratios with those characteristic of drag-optimized, flat-top configurations. The analysis is omitted for the sake of brevity, since it involves only a slight modification of that presented in Ref. 4. As expected, the lift-to-drag ratio of a minimum drag configuration is lower than that of a maximum lift-to-drag ratio configuration. The relative difference depends on the similarity parameter and, in the range of values considered in Figs. 5 through 12, is less than 8% if the length and the thickness are given, 23% if the length and the volume are given, and 9% if the thickness and the volume are given.

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Fig. 4 Maximum lift-to-drag ratio.

Fig. 5 Optimum power law exponent.

Fig. 6 Maximum lift-to-drag ratio.

Fig. 7 Optimum power law exponent.

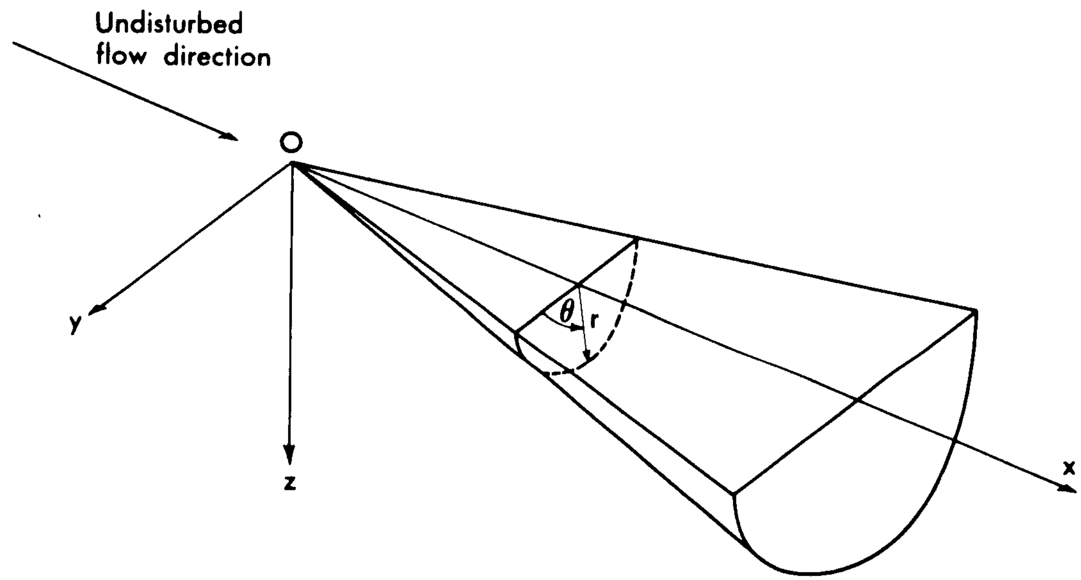
Fig. 8 Optimum thickness ratio.

Fig. 9 Maximum lift-to-drag ratio.

Fig. 10 Optimum power law exponent.

Fig. 11 Optimum thickness ratio.

Fig. 12 Maximum lift-to-drag ratio.



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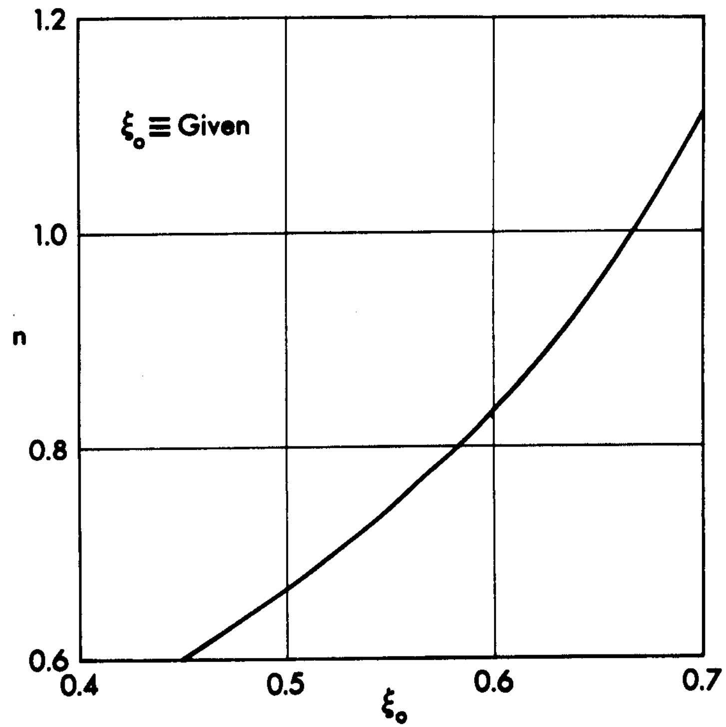

Fig. 2 Power law exponent.

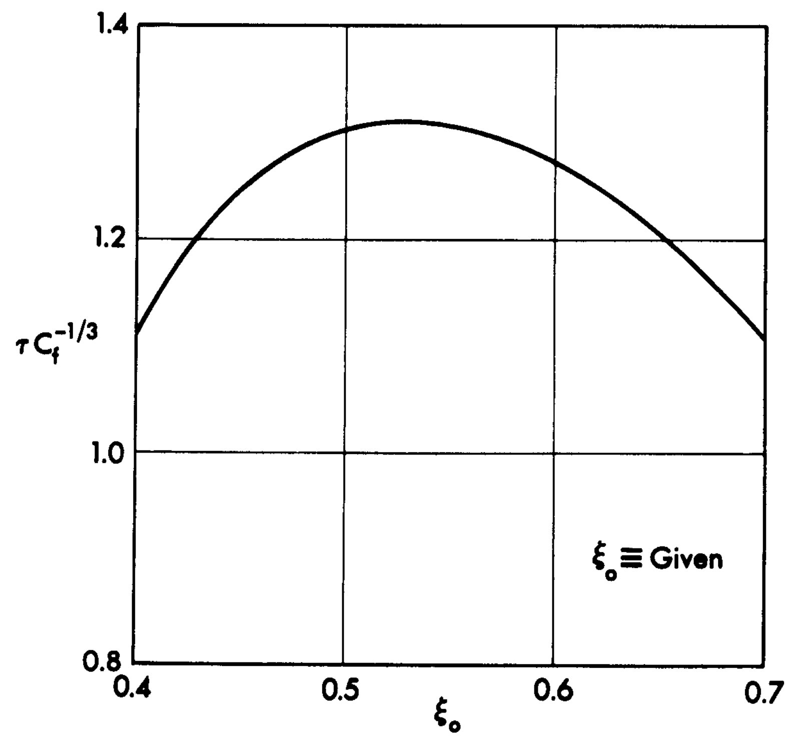

Fig. 3 Optimum thickness ratio.

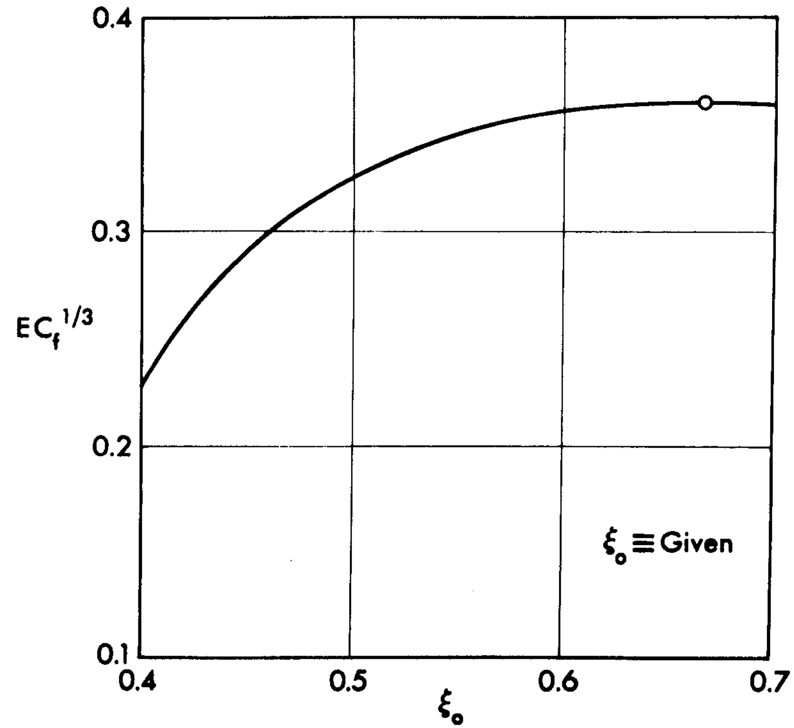

Fig. 4 Maximum lift-to-drag ratio.

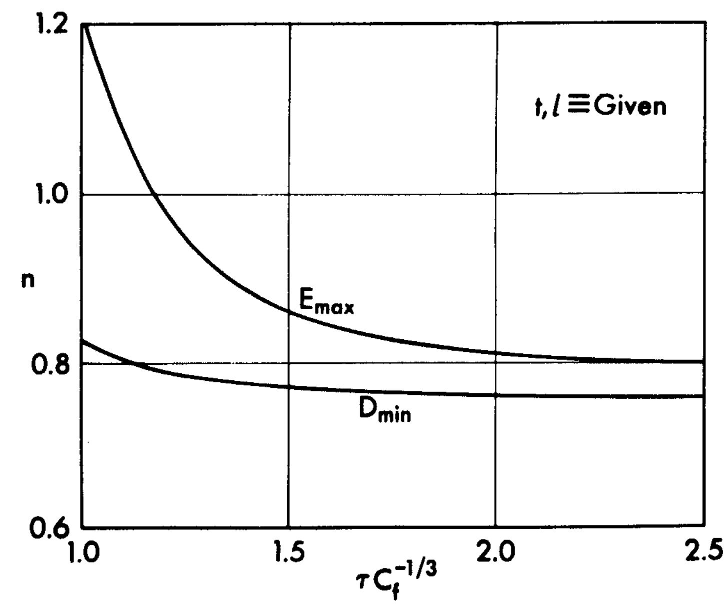

Fig. 5 Optimum power law exponent.

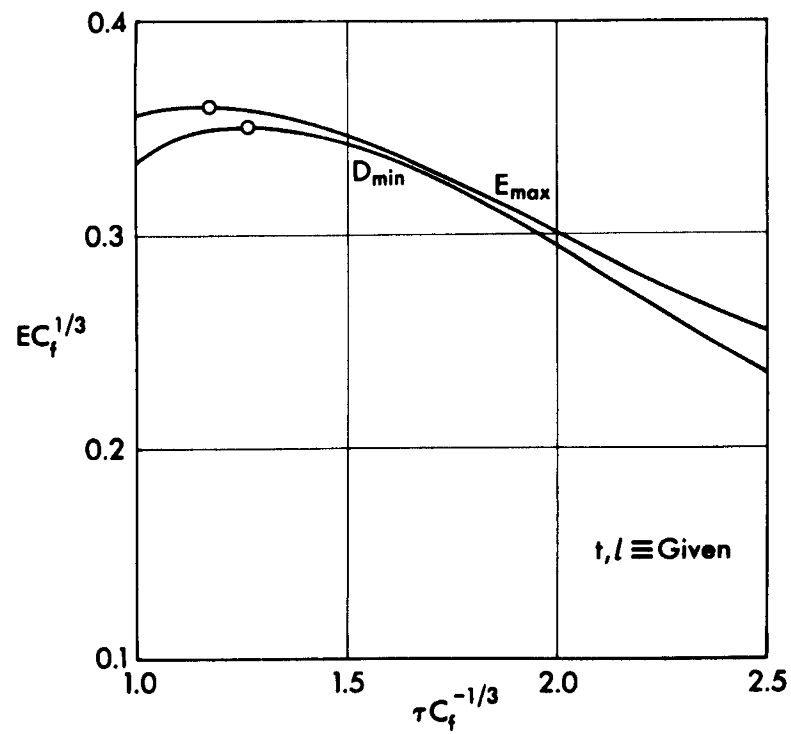

Fig. 6 Maximum lift-to-drag ratio.

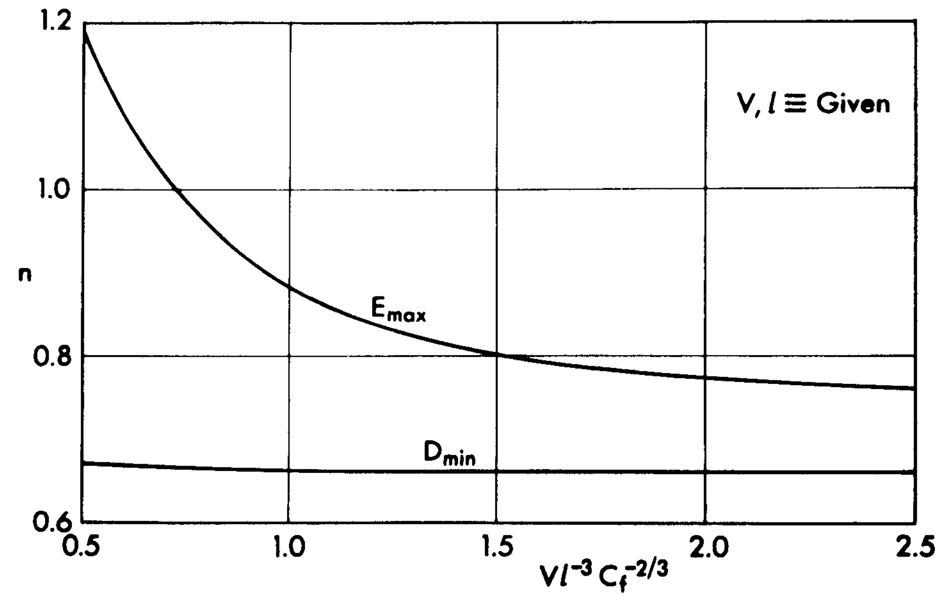

Fig. 7 Optimum power law exponent.

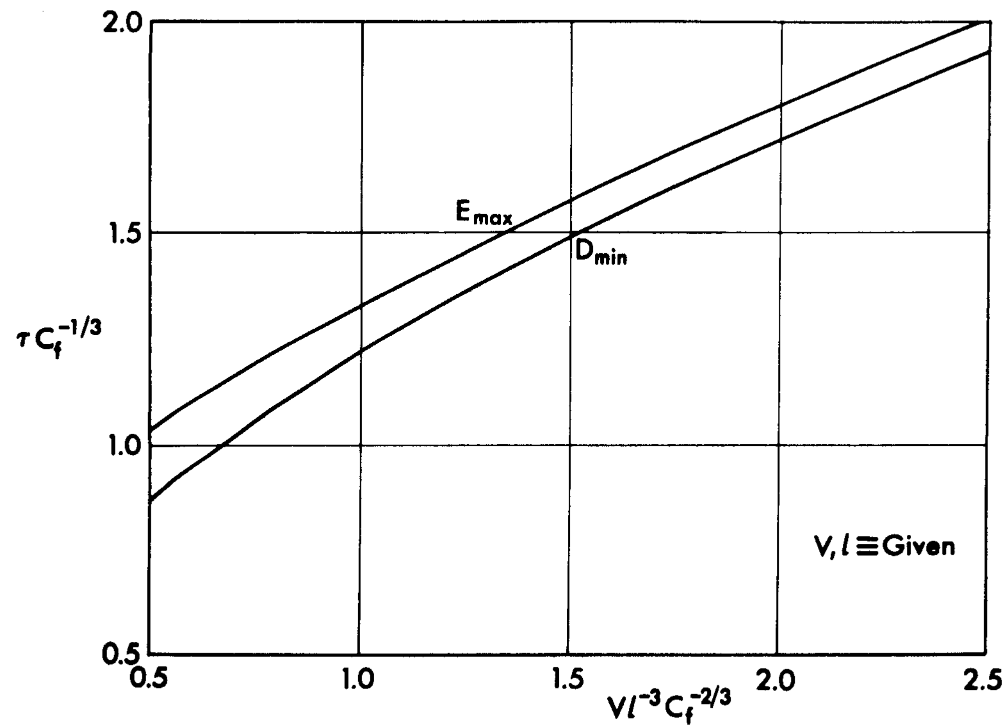

Fig. 8 Optimum thickness ratio.

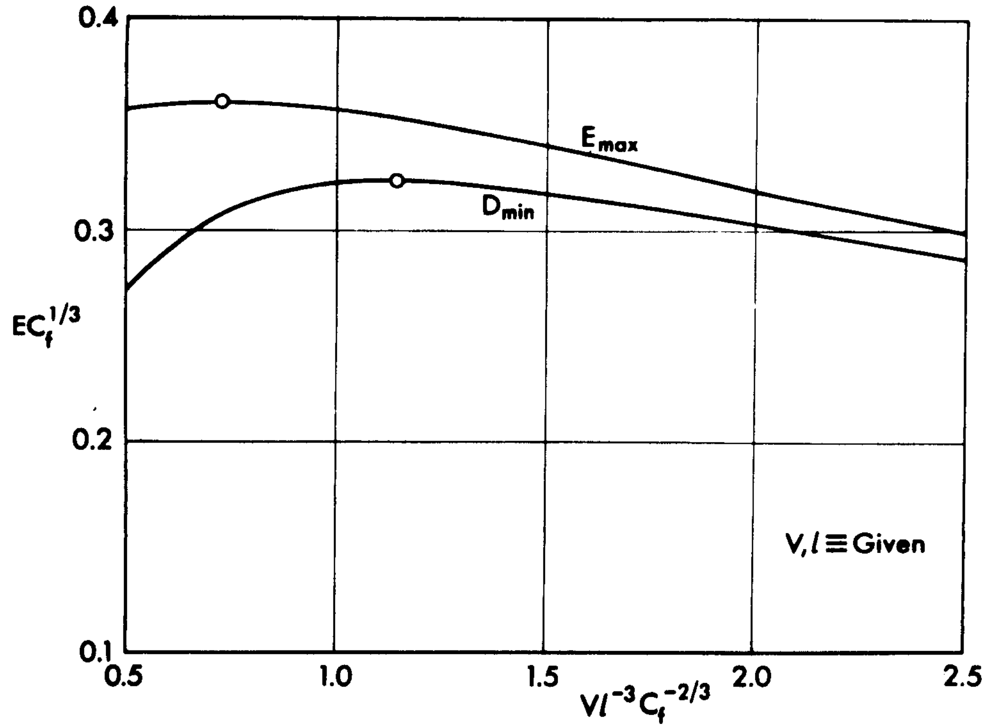

Fig. 9 Maximum lift-to-drag ratio.

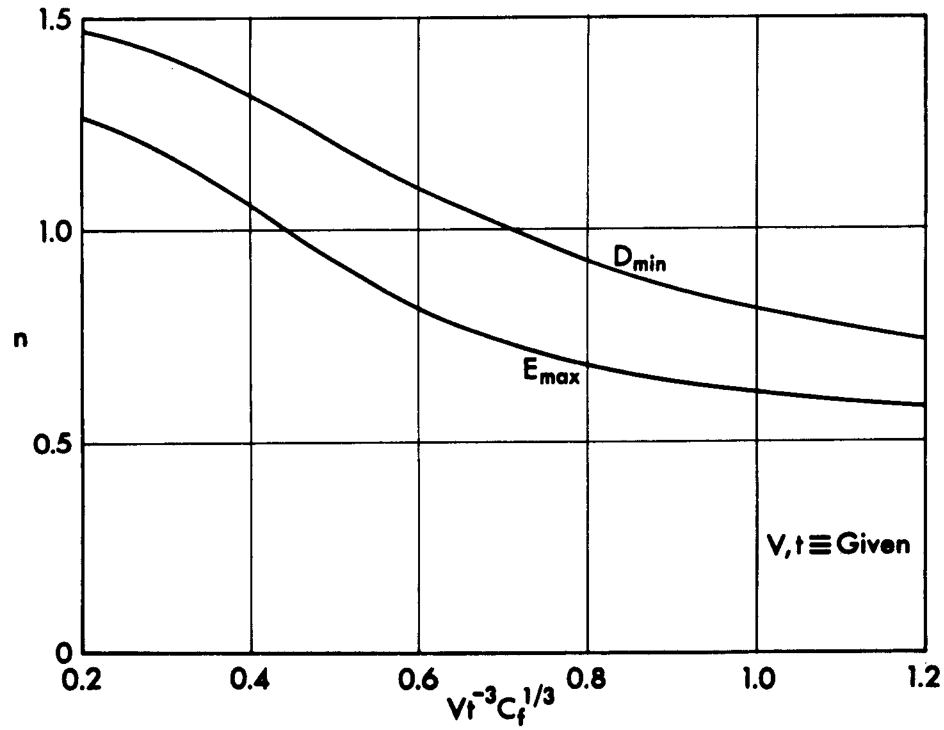

Fig. 10 Optimum power law exponent.

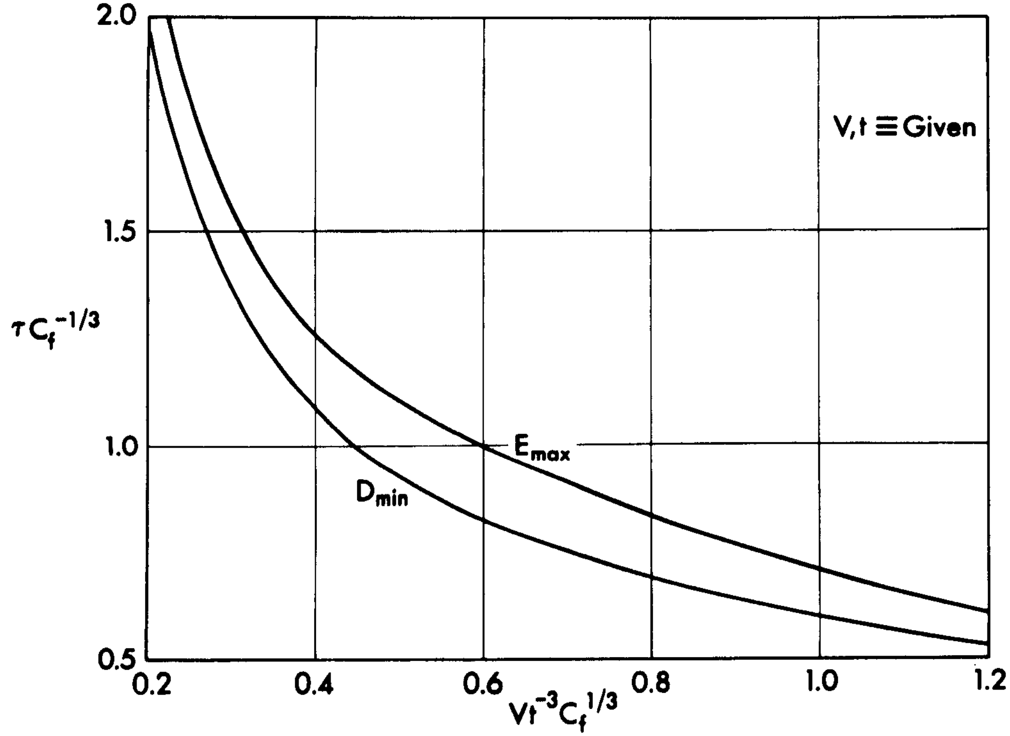

Fig. 11 Optimum thickness ratio.

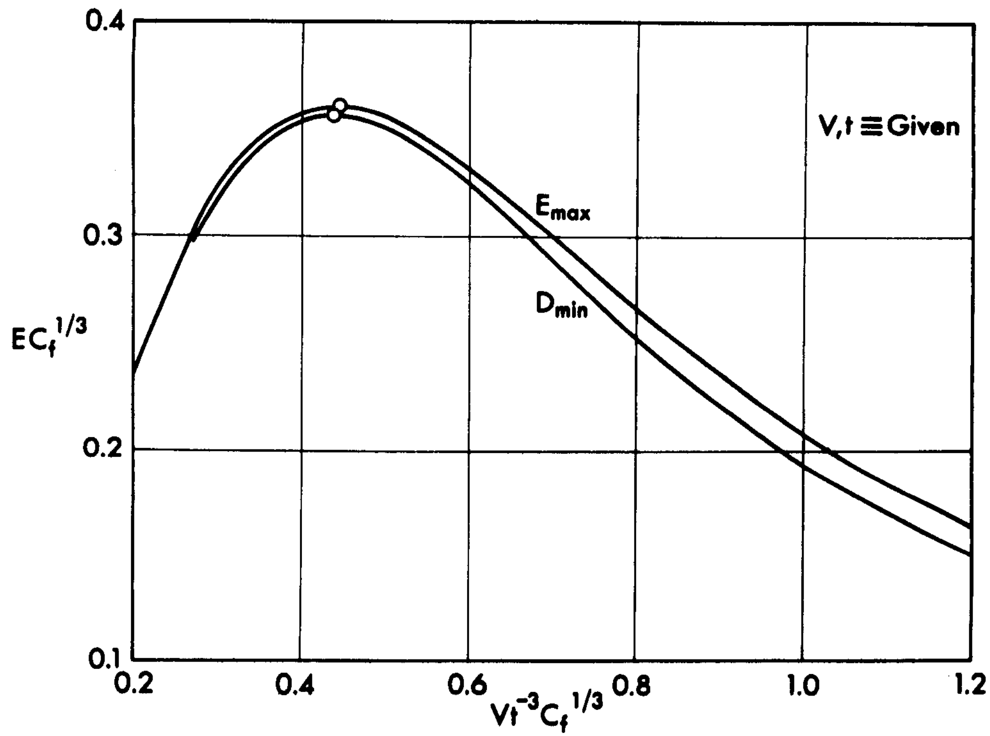

Fig. 12 Maximum lift-to-drag ratio.